

Phi – The Golden Number

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What makes a single number so interesting that ancient Greeks, Renaissance artists and 21st century novelists would all write about it? This “golden” number, now known as “Phi,” was written about by Euclid in “Elements” around 300 B.C., by Luca Pacioli, a contemporary of Leonardo Da Vinci, in “De Divina Proportione” in 1509 and by Dan Brown in 2003 in his best selling novel, “The Da Vinci Code.” With the movie release of the “The Da Vinci Code” on May 19, the quest to know Phi will be brought even more into the mainstream of pop culture, particularly with powerhouses Tom Hanks in the lead role and Ron Howard as director. The allure of “The Da Vinci Code” is that it creatively integrates fiction with both fact and myth from art, history, theology and mathematics, leaving the reader never really knowing what is truth and what is not. In this article, we’ll discuss this golden number Phi, and its mathematical cousin, the Fibonacci series, both of which have roles in the plot of this murder mystery, and distinguish between the myth and the math.

Known as the Divine Proportion and the Golden Section, Mean or Ratio, Phi truly is unique in its mathematical properties and pervasive in its appearance throughout nature. The “mathematically challenged” may be more interested in the appearances of Phi in nature, its application to art, architecture and design, and its potential for insights into the spiritual realm, but let’s begin with the purest of facts about Phi, which are found in mathematics. Most everyone learned about the number Pi in their school days, but relatively few curriculums included Phi, perhaps for the very reason that grasping all its manifestations often takes one beyond the academic into the realm of the spiritual just by the simple fact that Phi unveils a constant of design that applies to so many aspects of life. Both Pi and Phi are irrational numbers with an infinite number of digits after the decimal point, as indicated by “...”, the ellipsis. Where Pi or π (3.14...) is the ratio of the circumference of a circle to its diameter, Phi or ϕ (1.618...) is the ratio that results when a line is divided in one very special and unique way. To illustrate, suppose you were asked to take a string and cut it. There’s any number of places that you could cut it, and each place would result in different ratios for the length of the small piece to the large piece, and of the large piece to the entire string. There is one unique point, however, at which ratio of the large piece to the small piece is exactly the same as the ratio of the whole string to the large piece, and at this point the ratio of both is 1.618 to 1, or Phi.

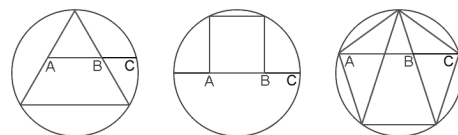


What makes this so much more than an interesting exercise in mathematics is that this proportion appears throughout creation and extensively in the human face and body. It’s found in the proportions of many other animals, in plants, the solar system, and even in the price and timing movements of stock markets. Its appeal thus ranges from mathematicians to naturalists to artists to mystics.

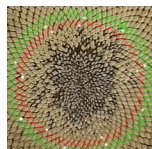
Part of the uniqueness of Phi is that it can be derived in many other ways than segmenting a line. Phi is the only number whose square is greater than itself by one, expressed mathematically as $\phi^2 = \phi + 1 = 2.618$. Phi is also the only number whose reciprocal is less than itself by one, expressed as $1/\phi = \phi - 1 = 0.618$. Where 1.618 is represented in upper case as Phi or ϕ , its near twin or reciprocal, 0.618, is often represented in lower case as phi or ϕ .

The Fibonacci series, also a plot element in “The Da Vinci Code,” provides yet another way to derive Phi mathematically. The series is quite simple. Start with 0 and add 1 to get 1. Then repeat the process of adding each two numbers in the series to determine the next one: 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, and so on. The relationship to Phi is found by dividing each number by the one before it. The further you go in the series, the closer the result gets to Phi. For example, while $5/3 = 1.666$ and $13/8 = 1.625$, go further into the series and you’ll find that $233/144 = 1.61805$, a very close approximation of Phi, which to ten decimal places is 1.6180339887.

Phi is also found in geometry, appearing in basic constructions of an equilateral triangle, square and pentagon placed inside a circle, as well as in more complex three-dimensional solids such as dodecahedrons, icosahedrons and “Bucky balls,” which were named for Buckminster Fuller and are the basis for the shapes of both Carbon 60 and soccer balls. In each of the images shown, AB has a length of 1 and is the golden section, or Phi point, of AC, which has a length of 1.618 or Phi.



Golden sections appearing in basic geometric shapes

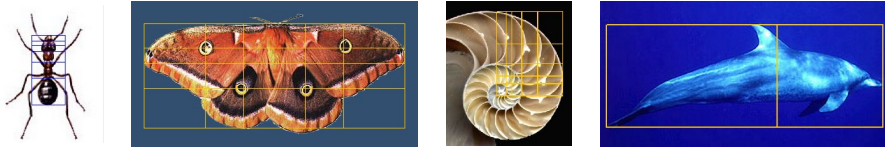


Fibonacci spirals in a sunflower seed, with 55 clockwise spirals in red and 89 counter-clockwise spirals in green.

There are many other fascinating mathematical relationships and oddities in both Phi and the Fibonacci series that can be explored in more depth, but for now let’s now take a step away from the purely mathematical and venture into nature, where Phi and the Fibonacci series manifest themselves pervasively, but not universally.

Fibonacci numbers frequently appear in the numbers of petals in a flower. Lilies have 3, buttercups have 5 and delphiniums have 8. As you proceed to flowers with higher numbers of petals, we find that this number is most often a Fibonacci number, but not always. (Even clovers occasionally depart from their typical three leaf configuration to yield the lucky fourth.) Marigolds usually have 13 petals, black-eyed Susans have 21, pyrethrum have 33 and daisies have 55 or 89. Spirals in seed pods, such as in pine cones and sunflowers, appear as dual spirals in opposite directions, with the number of spirals being successive Fibonacci numbers.

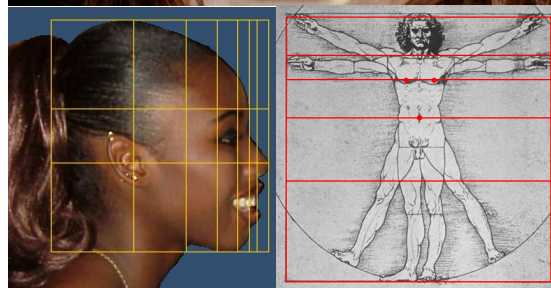
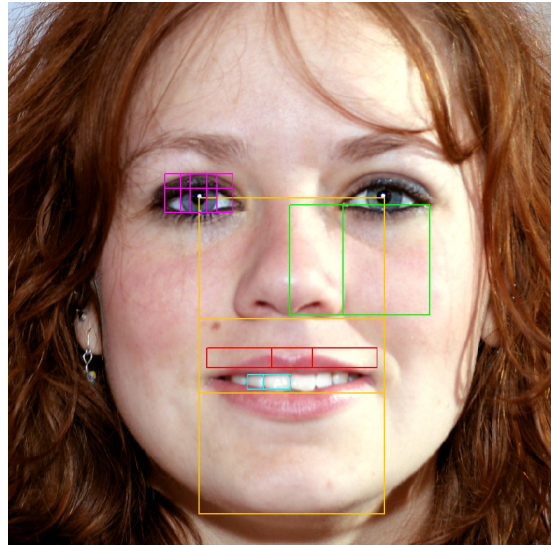
The positions and proportions of the key dimensions of many, if not most, animals are based on Phi. Examples include the body sections of ants and other insects, the wing dimensions and location of eye-like spots on moths, the spirals of sea shells and the position of the dorsal fins on porpoises. Even the spirals of human DNA embody phi proportions.



Every gridline created by PhiMatrix software is in a phi relationship (1.618:1) to the gridlines adjacent to it. This easily reveals the phi relationships that can be found in everything from DNA to the human face to stock markets.

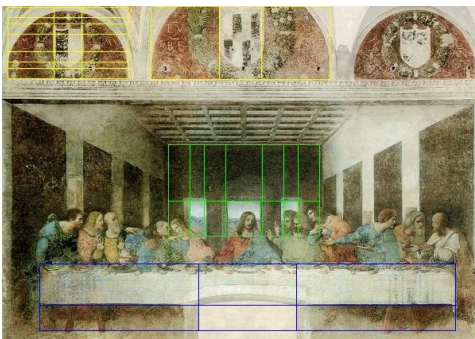
More intriguing yet is the extensive appearance of Phi throughout the human form and the impact that this has on our perceptions of human beauty. Some would argue that beauty is in the eye of the beholder, but there is sound basis in scientific study and evidence to support that what we perceive as beauty in both men and women is based on how closely the proportions of facial and body dimensions come to Phi. It seems that Phi is hard-wired into our consciousness as a guide to beauty. For this reason, Phi is applied in both facial plastic surgery (beautyanalysis.com) and cosmetic dentistry (phidental.com) as a guide to achieving the most natural and beautiful results in facial features and appearance. Note in the illustrations how Phi appears again and again in the design of the human face and body.

With all the unique mathematical properties of Phi and its appearance throughout creation, it's little wonder that mankind would not only take notice of this number, but also use it to capture the beauty and harmony of nature in its own creations. In some cases, mankind's application of Phi is undeniable. In other cases, it is still the subject of debate. The Great Pyramid of Egypt appears to embody Phi in the ratios of its base, height and hypotenuse, but its state of ruin and the absence of the mention of Phi in ancient Egyptian writings make it difficult to prove conclusively that this was by design and not coincidence. Since the Greeks knew of Phi at the time of the building of the Parthenon, it seems quite clear to many that the Phi relationships found therein were by design, yet the evidence is not complete and there are those who contest this as well. It is recognized that Leonardo Da Vinci used Phi, known in the 1500's as "The Divine Proportion," in a number of his paintings. While this is undeniable, some see Phi relationships where others do not believe they were intended. The dimensions of the treasured Stradivarius violins built around 1700 show clear Phi relationships. More modern applications of Phi can be seen in Notre Dame in Paris, the United Nations building in New York and the CN Tower in Toronto. It's even being used in high fashion clothing design in the "Phi Collection" announced in 2004 and covered by Vogue, Elle and Vanity Fair.



PhiMatrix software reveals golden sections throughout the human face and body in the positions and proportions of key defining features.

As if there were not already enough to bring wonder about Phi, even its pronunciation generates differences of opinion. Is it pronounced "fee" like "flee" or is it "fy" like "fly?" The answer is "both." In Greek, Phi is pronounced "fee," just as Pi is pronounced "pee." So, while Greeks and purists opt for the Greek pronunciation, most English and American dictionaries list the common usage of pronunciation as "fy" and "pie" for these venerable symbols of mathematics.



Golden sections throughout Da Vinci's "The Last Supper."

Phi continues to open new doors in our understanding of life and the universe. It appeared in Roger Penrose's discovery in the 1970's of "Penrose Tiles," which allowed surfaces to be tiled in five-fold symmetry, a task previously thought impossible. It appeared again in the 1980's in the three-dimensional molecular arrangement of quasi-crystals, a newly discovered form of matter. As we enter the 21st century, Phi seems to be having a rebirth in integrating knowledge across a wide variety of fields of study.

The description of this proportion as Divine is perhaps still fitting because it is seen by many as a door to a deeper understanding of beauty and spirituality in life. That's an incredible role for a single number to play, but then again this one number has played an incredible role in human history and in the foundations of life itself. The line between its

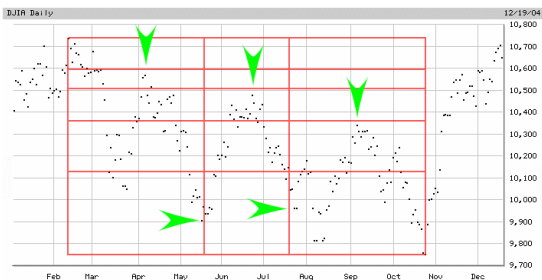
mathematical and mystical aspects is thus not easily drawn. Phi does not appear explicitly in the Bible or other ancient scriptures, yet we find that the dimensions given by God to Noah for the Ark and to Moses for the Ark of the Covenant both reflect a 5 to 3 proportion, Fibonacci numbers with a ratio of 1.666, and a reasonably close approximation to Phi. The trigonometric sine of the number 666, the sign of the beast in the Bible, is exactly $-\frac{\sqrt{2}}{2}$, or -0.0809. Curiously enough, even the symbol for Phi, a circle with a line drawn through it, can be thought to represent a zero, or void, divided by one, or Unity, to create beauty, analogous to God creating the universe from nothing.

In matters of reason, seeing is believing but in matters of faith, it is believing first that opens the door to seeing. Just as we need two eyes to add depth to our perception in vision, both faith and reason serve us in adding depth to our understanding of life and the universe in which we live. The best way to know for yourself where Phi is present and where it is imagined is to explore, learn and reach your own conclusions. You can learn more about Phi at www.goldennumber.net and download PhiMatrix software at www.phimatrix.com to easily discover and apply Phi proportions for yourself, as shown in the illustrations in this article. Enjoy the “phi”nomemon, whether to enhance your own understanding and appreciation of beauty and harmony in life or to apply it to your own artistic creations, like Leonardo Da Vinci and other masters before you.

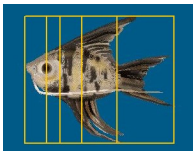
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Phi ratios in the fuselage to tail dimensions of a 767



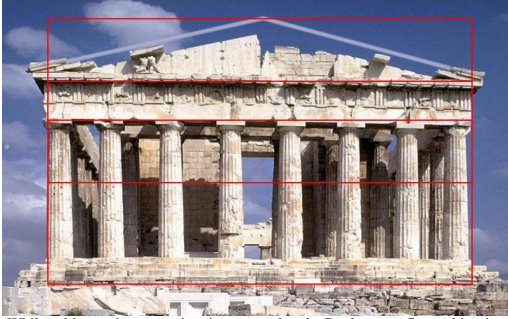
Phi ratios in the timing and price of high and lows in the 2004 Dow Jones Industrial Average.



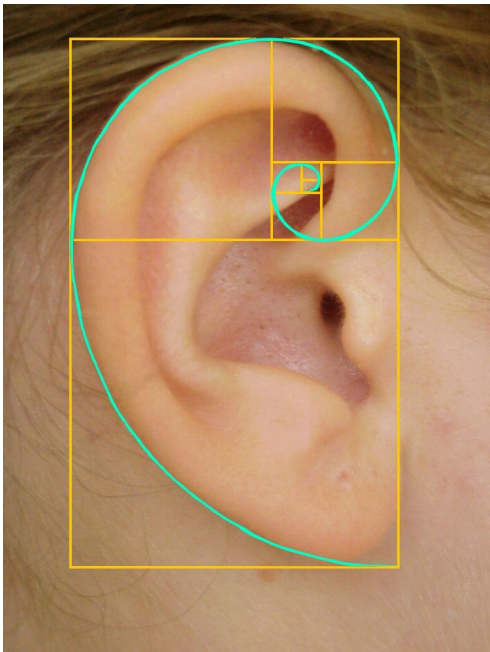
Phi defines the position of the tail, fin and eye of an angel fish.



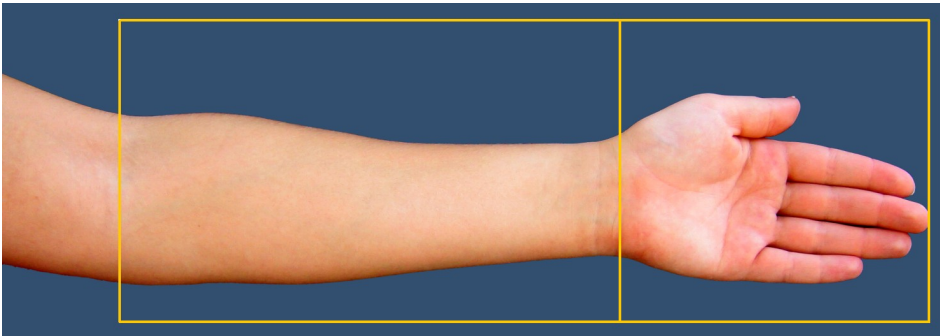
Phi defines the ratio of the neck to the body in a violin.



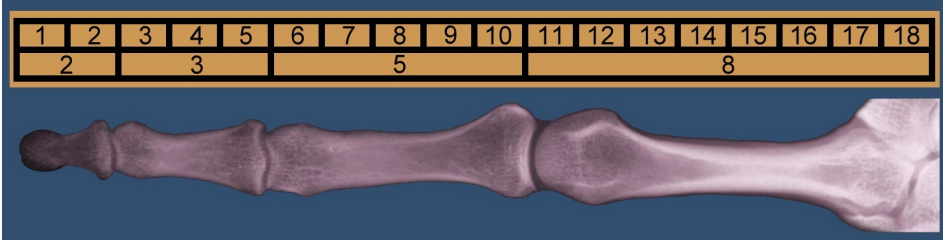
While subject to interpretation, it appears that the Parthenon reflects phi ratios in the height of the columns and face in relation to its height.



A spiral derived from a golden rectangle based on phi defines the spiral of this ear.



The dividing point at the base of the hand is a golden section of the full length of this forearm.



The sections of the human finger reflect the Fibonacci numbers 2, 3, 5 and 8 in their successive lengths.