

Figure 22

As noted above, the icosahedron and the dodecahedron are intimately related to the Golden Ratio, in more ways than one. For example, the twelve vertices of any icosahedron can be divided into three groups of four, with the vertices of each group lying at the corners of a *Golden Rectangle* (a rectangle in which the ratio of length to width is the Golden Ratio). The rectangles are perpendicular to each other, and their one common point is the center of the icosahedron (Figure 22). Similarly, the centers of the twelve pentagonal faces of the dodecahedron can be divided into three groups of four, and each of those groups also forms a Golden Rectangle.

The close associations between some plane figures, such as the pentagon and the pentagram, and some solids, such as the Platonic solids, and the Golden Ratio lead to the inescapable conclusion that the Greek interest in the Golden Ratio probably started with attempts to construct such plane figures and solids. Most of this mathematical effort occurred around the beginning of the fourth century B.C. There exist, however, numerous claims that the Golden Ratio is embodied in the architectural design of the Parthenon, which was built and decorated between 447 and 432 B.C., under the rule of Pericles. Can these claims be verified?

The close associations between some plane figures, such as the pentagon and the pentagram, and some solids, such as the Platonic solids, and the Golden Ratio lead to the inescapable conclusion that the Greek interest in the Golden Ratio probably started with attempts to construct such plane figures and solids. Most of this mathematical effort occurred around the beginning of the fourth century B.C. There exist, however, numerous claims that the Golden Ratio is embodied in the architectural design of the Parthenon, which was built and decorated between 447 and 432 B.C., under the rule of Pericles. Can these claims be verified?

### THE VIRGIN'S PLACE

The Parthenon ("the virgin's place" in Greek) was built on the Acropolis in Athens as a temple sacred to the cult of Athena Parthenos (Athena the Virgin). The architects were Ictinus and Callicrates, and Phidias and his assistants and students were charged with supervising the sculptures. Sculptured groups ornamented the pediments terminating the roof at the eastern and western ends. One group depicted the birth of Athena and the other the contest between Athena and Poseidon.

Somewhat deceptive in its simplicity, the Parthenon remains one

of the finest architectural expressions of the ideal of clarity and unity. On September 26, 1687, Venetian artillery hit the Parthenon directly, during an attack on the Ottoman Turks who held Athens at the time and who used the Parthenon as a powder magazine. While the damage was extensive, the basic structure remained intact. In describing this event, General Konigsmark, who accompanied the field commander, wrote: "How it dismayed His Excellency to destroy the beautiful temple which had existed three thousand years!" Numerous attempts have been made, especially since the end of the Turkish control (in 1830), to discover some mathematical or geometrical basis supposedly employed to achieve the Parthenon design's high perfection. Most books on the Golden Ratio state that the dimensions of the Parthenon, while its triangular pediment was still intact, fit precisely into a Golden Rectangle. This statement is usually accompanied by a figure similar to that in Figure 23. The Golden Ratio is assumed to feature in other dimensions of the Parthenon as well. For example, in one of the most extensive works on the Golden Ratio, Adolph Zeising's *Der Goldne Schnitt*

(The golden section; published in 1884), Zeising claims that the height of the façade from the top of its tympanum to the bottom of the pedestal below the columns is also divided in a Golden Ratio by the top of the columns. This statement was repeated in many books, such as Matila Ghyka's influential *Le Nombre d'or* (The golden number; appeared in 1931). Other authors, such as Miloutine Borissavlievitch in

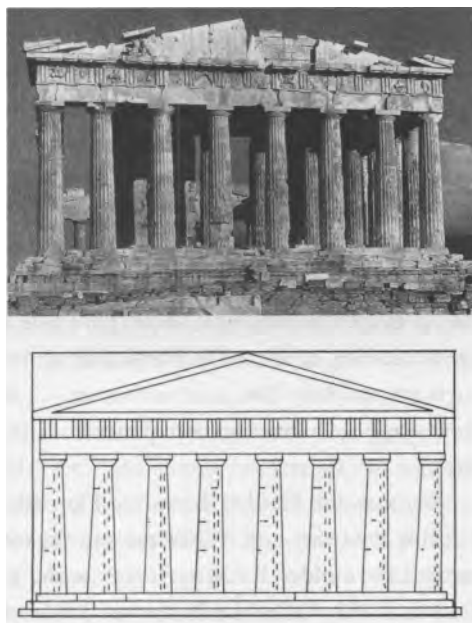


Figure 23

*The Golden Number and the Scientific Aesthetics of Architecture* (1958), while not denying the presence of  $\phi$  in the Parthenon's design, suggest that the temple owes its harmony and beauty more to the regular rhythm introduced by the repetition of the same column (a concept termed the "law of the Same").

The appearance of the Golden Ratio in the Parthenon was seriously questioned by University of Maine mathematician George Markowsky in his 1992 *College Mathematics Journal* article "Misconceptions about the Golden Ratio." Markowsky first points out that invariably, parts of the Parthenon (e.g., the edges of the pedestal; Figure 23) actually fall outside the sketched Golden Rectangle, a fact totally ignored by all the Golden Ratio enthusiasts. More important, the dimensions of the Parthenon vary from source to source, probably because different reference points are used in the measurements. This is another example of the number-juggling opportunity afforded by claims based on measured dimensions alone. Using the numbers quoted by Marvin Trachtenberg and Isabelle Hyman in their book *Architecture: From Prehistory to Post-Modernism* (1985), I am not convinced that the Parthenon has anything to do with the Golden Ratio. These authors give the height as 45 feet 1 inch and the width as 101 feet 3.75 inches. These dimensions give a ratio of width/height of approximately 2.25, far from the Golden Ratio of 1.618 . . . Markowsky points out that even if we were to take the height of the apex above the pedestal upon which the series of columns stands (given as 59 feet by Stuart Rossiter in his 1977 book *Greece*), we still would obtain a width/height ratio of about 1.72, which is closer to but still significantly different from the value of  $\phi$ . Other researchers are also skeptical about  $\phi$ 's role in the Parthenon's design. Christine Flon notes in *The World Atlas of Architecture* (1984) that while "it is not unlikely that some architects . . . should have wished to base their works on a strict system of ratios . . . it would be wrong to generalize."

So, was the Golden Ratio used in the Parthenon's design? It is difficult to say for sure. While most of the mathematical theorems concerning the Golden Ratio (or "extreme and mean ratio") appear to have been formulated after the Parthenon had been constructed, considerable knowledge existed among the Pythagoreans prior to that. Thus, the

Parthenon's architects might have decided to base its design on some prevailing notion for a canon for aesthetics. However, this is far less certain than many books would like us to believe and is not particularly well supported by the actual dimensions of the Parthenon.

Whether or not the Golden Ratio features in the Parthenon, what is clear is that whichever mathematical "programs" concerning the Golden Ratio were instituted by the Greeks in the fourth century B.C., that work culminated in the publication of Euclid's *Elements*, in around 300 B.C. Indeed, from a perspective of logic and rigor, the *Elements* was long thought to be an apotheosis of certainty in human knowledge.

### EXTREME AND MEAN RATIO

In 336 B.C., twenty-year-old Alexander (the Great) of Macedonia succeeded to the throne and, by a sequence of brilliant victories, conquered most of Asia Minor, Syria, Egypt, and Babylon and became ruler of the Persian Empire. A few years before his death at the young age of thirty-three, he founded what became the greatest monument to his name—the city of Alexandria near the mouth of the Nile.

Alexandria was located at the crossroads of three great civilizations: Egyptian, Greek, and Jewish. Consequently, it became an extraordinary intellectual center that lasted for centuries and the birthplace of such remarkable achievements as the Septuagint (meaning "translation of the 70")—the Greek translation of the Old Testament, traditionally attributed to seventy-two translators. The translation was begun in the third century B.C., and the work progressed in several stages over about a century.

After Alexander's death, Ptolemy I gained control over Egypt and the African dominions by 306 B.C., and among his first actions was the establishment of the equivalent of a university (known then as the Museum) in Alexandria. This institution included a library, which, following an immense gathering effort, was reputed to hold at one time 700,000 books (some confiscated from unlucky tourists). The first staff of teachers at the Alexandria school included Euclid, the author of the